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## DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

### SOLUTIONS OF PROBLEMS.

37. Proposed by A. H. BELL, Box 184, Hillsboro, Illinois.

Find the first four integral values of  $n$  in  $\frac{n(5n-3)}{2} = \square$ .

I. Solution by the PROPOSER, and Prof. J. SCHEFFER, A. M., Hagerstown, Maryland.

Let the heptagonal numbers  $\frac{n(5n-3)}{2} = \square = y^2$ . Clearing of fractions, then multiplying by 20 and adding 9 to both sides,  $(10n-3)^2 = 40y^2 + 9 = \square = x^2$ .  $\therefore n = (x+3)/10 \dots \dots (1)$ . Let  $x^2 - 40y^2 = 9$  be written  $3^2 x_1^2 - 40 \cdot 3^2 y_1^2 = 3^2$ . Dividing by  $3^2$  and solving  $x_1^2 - 40y_1^2 = 1$ , the convergent of  $\sqrt{40}$  is  $19/3$ .  $\therefore x_1 = 19$ ; by the general formula  $x_{n+1} = 2x_1 \times x_n - x_{n-1}$ , we have  $x_1 = 1, 19, 721, 27379, 1039681, 39, 480, 499$ , etc. As  $x = 3x_1$ , and as integral values for  $n$  can only be obtained by the numbers ending in 9, then in (1)  $n = 1, 6, 8214$ , and  $11844150$ .

II. Solution by JOSIAH H. DRUMMOND, LL. D., Portland, Maine.

The expression readily reduces to  $10n^2 - 6n = \square \dots \dots (1)$ . It is readily seen that  $n=1$  satisfies this equation. Take  $n=m+1$ , substitute it in (1), reduce and we have  $10m^2 + 14m + 4 = \square = (\text{say}) (pm-2)^2$ , from which we obtain  $m = (4p+14)/(p^2-10)$ . Take  $p=4$  and we have  $m=5$ , and  $n=6$ , the second value. Now take  $n=m+6$ , substitute in (1) and reduce as before and we find,  $m=43$ , and  $n=49$ , the third value. In  $(4p+14)/(p^2-10)$  take  $p=19/6$ ,  $p^2-10=1/36$  and we have  $m=960$ , and  $n=961$ , the fourth value.

III. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

If we put the expression equal  $x^2$  and reduce, we readily obtain  $10n = 3 \pm \sqrt{40x^2 + 9}$ . Putting  $x=1, 2, 9, 40$  and  $77$ , respectively, I find the first four integral values of  $n$  to be, respectively,  $\pm 1, 6, -25$ , and  $49$ .

38. Proposed by H. C. WILKES, Skull Run, West Virginia.

Let  $n$  be any number and let  $n^3 + 1 = x$ .

Then  $x^3 + (2x-3)^3 + (nx-3n)^3 = n^3 x^3$ . Demonstrate.

I. Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics in New Windsor College, New Windsor, Maryland.

The simplest way is to substitute the value of  $x$  and expand. An identity is the result.

II. Solution by ARTEMAS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

Substituting  $n^3 + 1$  for  $x$ ,  $(n^3 + 1)^3 + (2n^3 - 1)^3 + (n^4 - 2n)^3 = (n^4 + n)^3$ ;